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Computation of fuzzy eigenvalues and fuzzy eigenvectors for the correspondence analysis in personal selection

Informatics and automatics

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The second approach, extending and developing correspondence analysis with fuzzy numbers, is used. This extension, due to its complex and difficult character, can be regarded as a sequence of phases of development, leading from mathematical principles to the developing of appropriate computational and graphical software. Here, authors confine ourselves to laying down some of the mathematical principles, and by necessity our examples will be simple, for illustrative purposes. Authors apply fuzzy eigenvalue approach for the correspondence analysis in personal selection. In this article, we employ a method for the treatment of the fuzzy eigenvalue problem of fuzzy correspondence analysis, which is based on a two-step analysis.

Key words: fuzzy numbers, correspondence analysis, fuzzy matrix, fuzzy eigenvalue, multicriteria, fuzzy eigenvector.

Introduction

A basic preference of correspondence analysis over other methods falling joint graphical displays is that it creates two double displays, whose row and column geometries have similar explanations; in other multivariate approaches to graphical data representation, this duality does not exist. It can be represented as a way of determining the best concurrent representation of two data sets that consist of the rows and columns of a data matrix. Moreover, it can determine nonlinear relationships between rows and columns of a multidimensional contingency table. Elements of correspondence analysis can be traced back to works of K. Pearson, C. Spearman, R. Fisher, L. Guttman, C. Hayashi and others. However, its final conclusive form was developed by Jean-Paul Benzecri [2, 4].

The basic application of correspondence analysis, and its expanding to multiple correspondence analysis as well, can be done in different ways using some types of multidimensional data matrices. There are, however, many examples of application, where the use of fuzzy numbers is unavoidable. The processing of such data, by using the standard correspondence analysis is not natural or even useful. The introduction of fuzzy quantities, such as fuzzy numbers, fuzzy matrices, fuzzy vectors and spaces, fuzzy eigenvalues and eigenvectors, seems to be essential [12].

Next, we determine the most important works related to the fuzzy extension of correspondence analysis. This extension can be carried out in different ways. The extension of correspondence analysis to fuzzy correspondence analysis can be done mainly in the following two ways – fuzzy data analysis of crisp numbers and fuzzy data analysis of fuzzy numbers. In correspondence analysis, the first case essentially corresponds to the case in which we have quantitativecontinuous variables, and we wish to convert them into qualitative-discrete ones. This can be done by using either crisp or fuzzy partitions called fuzzy coding [6, 13]. Fuzzy coding can be employed to recode continuous data into ordered fuzzy categories. In the second approach we may consider contingency tables with fuzzy numbers. In this case we have to develop a special algebra in order to do the relative analysis which pertains to correspondence analysis [10]. The authors [3, 5, 8, 9, 13] follow mainly the first approach by using multiple correspondence analysis.

In this article, we are going to follow the second approach, expanding and developing correspondence analysis with fuzzy numbers. This extension, due to its complex and difficult nature, can be viewed as a series of stages of development, leading from mathematical foundations to the development of relevant computational and graphical software. Here, we shall restrict ourselves to laying down some of the mathematical foundations, and by necessity our examples will be simple, for illustrative goals.

Definition 1. Let a two-way ($n \times p$) contingency table K, where its elements (k_i^j) are nonnegative fuzzy numbers (some could be ordinary real numbers). Following the known procedure of standard CA (transformation of initial fuzzy matrix K in the fuzzy matrix of profile-rows X, and calculation of the fuzzy matrices D and Q), but using the fuzzy arithmetical operations, the problem is reduced finally to finding the eigenvalues, eigenvectors of the fuzzy matrix,

$$\underset{p,p}{S} = X^T \otimes D \otimes X \otimes Q \tag{1}$$

This fuzzy extension of CA (Fuzzy Correspondence Analysis – FCA), algebraically leads to the study of the fuzzy eigenvalue problem for the fuzzy matrix S of FCA, which is to solve the fuzzy equation:

$$S \otimes u = \lambda \otimes u\,, \tag{2}$$

where

$$\underset{p,p}{S} = X^T \otimes D \otimes X \otimes Q = (s_i^j), \quad (i, j = 1, \ldots\ldots, p). \tag{3}$$

A fuzzy eigenvalue λ, of a square fuzzy matrix A is defined generally as a fuzzy number, [4, 7]. Solution of the fuzzy characteristic equation $A \otimes u = \lambda \otimes u$, where u is the corresponding fuzzy eigenvector, a non-zero fuzzy vector with fuzzy numbers as coordinates [1, 8].

Definition 2. The fuzzy eigenvalues of S are the fuzzy numbers λ, which are solutions of the fuzzy equation $S \otimes u = \lambda \otimes u$, where $S = (s_i^j)$, $u^T = (u_1, \ldots\ldots, u_p), 1 \le i,\ j \le p,$ and the s_i^j and u_i are fuzzy numbers. The fuzzy equation (2) according to the fuzzy arithmetic, is reduced to:

$$(s_i^1 \otimes u_1) \oplus \ldots \oplus (s_i^p \otimes u_p) = \lambda \otimes u_i \tag{4}$$

or using the Decomposition Theorem:

$$\left(\sum_{a \in [0,1]} a S^a\right) \otimes \left(\sum_{a \in [0,1]} a u^a\right) = \left(\sum_{a \in [0,1]} a \lambda^a\right) \otimes \left(\sum_{a \in [0,1]} a u^a\right) \tag{5}$$

Thus, the algebraic foundation of FCA leads to the fuzzy eigenvalue problem [11].

Definition 3. Taking the α-cuts of fuzzy equation we have an interval equation:

$$(S \otimes u)^a = (\lambda \otimes u)^a \Leftrightarrow S^a \otimes u^a = \lambda^a \otimes u^a\,, \quad a \in [0,1] \tag{6}$$

More specifically for the fuzzy matrix $S = (s_i^j)$ each α -cut $S^a = (s_i^j)^a$ α is interval matrix, i.e. its elements are ordinary closed intervals. That is,

$$S^a = (s_i^j)^a = [(s_i^j)_l^a, (s_i^j)_r^a] = [S_l^a, S_r^a]\,, \tag{7}$$

where the ordinary matrices $S_l^a = (s_i^j)_l^a$ and $S_r^a = (s_i^j)_r^a$ are the left and right boundary respectively, of the α-cut. Similarly,

$$\lambda^a = [\lambda_l^a, \lambda_r^a] \text{ and } u^a = [u_l^a, u_r^a] = [(u_i)_l^a, (u_i)_r^a] \tag{8}$$

Thus (13) is equivalent with

$$S^a \otimes u^a = \lambda^a \otimes u^a \iff [S_l^a, S_r^a] \otimes [u_l^a, u_r^a] = [\lambda_l^a, u_r^a] \otimes [u_l^a, u_r^a]$$

$$\Leftrightarrow \begin{bmatrix} [(s_1^1)_l^a, (s_1^1)_r^a] ... [(s_1^j)_l^a, (s_1^j)_r^a] ... [(s_1^p)_l^a, (s_1^p)_r^a] \\ ------------------- \\ [(s_i^1)_l^a, (s_i^1)_r^a] ... [(s_i^j)_l^a, (s_i^j)_r^a] ... [(s_i^p)_l^a, (s_i^p)_r^a] \\ ------------------- \\ [(s_p^1)_l^a, (s_p^1)_r^a] ... [(s_p^j)_l^a, (s_p^j)_r^a] ... [(s_p^p)_l^a, (s_p^p)_r^a] \end{bmatrix} \otimes \begin{bmatrix} [(u_1)_l^a, (u_1)_r^a] \\ ------ \\ [(u_i)_l^a, (u_i)_r^a] \\ ------ \\ [(u_p)_l^a, (u_p)_r^a] \end{bmatrix} = [\lambda_l^a, \lambda_r^a] \otimes \begin{bmatrix} [(u_1)_l^a, (u_1)_r^a] \\ ------ \\ [(u_i)_l^a, (u_i)_r^a] \\ ------ \\ [(u_p)_l^a, (u_p)_r^a] \end{bmatrix} \tag{9}$$

Definition 4. According to the two-steps method, first we calculate the interval matrix,

$$S^a = (s_i^j)^a = [(s_i^j)_l^a, (s_i^j)_r^a] = [S_l^a, S_l^a] = (1-\nu)S_l^a + \nu S_r^a, \qquad \nu \in [0,1] \tag{10}$$

$$S^a = [S_l^a, S_r^a] = [(s_i^j)_l^a, (s_i^j)_r^a] = [l_i^j + a(c_i^j - l_i^j), r_i^j - a(r_i^j - c_i^j)] = [l_i^j + aC_i^j, r_i^j - aR_i^j] \tag{11}$$

In the second step, we find the ordinary square matrices,

$$S_l^a = (s_i^j)_l^a = \begin{pmatrix} l_1^1 + C_1^1 a & .... & l_1^p + C_1^p a \\ \vdots & \ddots & \vdots \\ l_p^1 + C_p^1 a & .... & l_p^p + C_p^p a \end{pmatrix}, \tag{12}$$

and

$$S_r^a = (s_i^j)_r^a = \begin{pmatrix} r_1^1 - R_1^1 a & .... & r_1^p - R_1^p a \\ \vdots & \ddots & \vdots \\ r_p^1 - R_p^1 a & .... & r_p^p - R_p^p a \end{pmatrix}, \tag{13}$$

where $C_i^j = (c_i^j - l_i^j) \geq 0$ and $R_i^j = (r_i^j - c_i^j) \geq 0$, (for all *i*, *j*).

In ordinary correspondence analysis, the crisp matrix $S = X^T DXQ$ has eigenvalues $\lambda_s \in [0, 1]$, with max $\{\lambda_s\} = \lambda_1 = 1$. Therefore in FCA, the crisp matrices

$$S_l^a = (s_i^j)_l^a = (X^T \otimes D \otimes X \otimes Q)_l^a = (X^T)_l^a D_l^a X_l^a Q_l^a \tag{14}$$

and

$$S_r^a = (s_i^j)_r^a = (X^T \otimes D \otimes X \otimes Q)_r^a = (X^T)_r^a D_r^a X_r^a Q_r^a \tag{15}$$

are constructed under the same conditions as in ordinary CA, they have entries functions of a $\in$ [0, 1], and thus will have also crisp eigenvalues functions of *a*, $(\lambda_s)_l^a$ and $(\lambda_s)_r^a$, respectively.

Statement of the problem

The proposed method of the fuzzy AHP was applied for the personal selection problem, which is a multi-criteria decision making (MCDM) problem since it contains different and conflicting criteria. Suppose that an MCDM problem involves 4 criteria – C_1, C_2, C_3, C_4 and 4 alternatives. C_1 – Relevant experience; C_2 – Education; C_3 – Technical skills; C_4 – Relocation (Table).

Table. Linguistic performance rating of fuzzy numbers

	C_1	C_2	C_3	C_4
C_1	(1;1;1)	(6;7,8)	(2;3;4)	(4;5;6)
C₂	(1/8;1/7;1/6)	(1;1;1)	(1/6;1/5;1/4)	(1/4;1/3;1/2)
C_3	(1/4;1/3;1/2)	(4;5;6)	(1;1;1)	(2;3;4)
C_4	(1/6;1/5;1/4)	(2;3;4)	(1/4;1/3;1/2)	(1;1;1)

When the elements compared are expressed in the same units of measurement and we use a suitably precise measuring tool, then pairwise comparison is unnecessary. The answer to the question of how much more important one element is than the others is given simply by the combination of the measurements of all the elements. It is worth investigating the properties of the comparison matrix in the case of precise measurements.

Solution of the problem

Let us now take the initial contingency table *K* of FCA, to be precisely as follows:

$$S^a = [S_l^a, S_r^a] = [(s_i^j)_l^a, (s_i^j)_r^a] = [l_i^j + a(c_i^j - l_i^j), r_i^j - a(r_i^j - c_i^j)]$$

For some values of $a \in [0, 1]$, we give:

For the left side

$$S_l^a = \begin{bmatrix} 1 & 6+a & 2+a & 4+a \\ 0.015a+0.125 & 1 & 0.17+0.03a & 0.25+0.08a \\ 0.25+0.08a & 4+a & 1 & 2+a \\ 0.17+0.03a & 2+a & 0.25+0.08a & 1 \end{bmatrix}$$

If a=0 then $S_l^0 = \begin{bmatrix} 1 & 6 & 2 & 4 \\ 0.125 & 1 & 0.17 & 0.25 \\ 0.25 & 4 & 1 & 2 \\ 0.17 & 2 & 0.25 & 1 \end{bmatrix}$

If a=1 then $S_l^1 = \begin{bmatrix} 1 & 7 & 3 & 5 \\ 0.14 & 1 & 0.2 & 0.33 \\ 0.33 & 5 & 1 & 3 \\ 0.2 & 3 & 0.33 & 1 \end{bmatrix}$

For the right side

$$S_r^a = \begin{bmatrix} 1 & 8-a & 4-a & 6-a \\ 0.17-0.03a & 1 & 0.25-0.05a & 0.5-0.17a \\ 0.5-0.17a & 6-a & 1 & 4-a \\ 0.25-0.05a & 4-a & 0.5-0.17a & 1 \end{bmatrix}$$

If $a = 0$ then $S_r^0 = \begin{bmatrix} 1 & 8 & 4 & 6 \\ 0.17 & 1 & 0.25 & 0.5 \\ 0.5 & 6 & 1 & 4 \\ 0.25 & 4 & 0.5 & 1 \end{bmatrix}$

If $a = 1$ then $S_r^1 = \begin{bmatrix} 1 & 7 & 3 & 5 \\ 0.14 & 1 & 0.2 & 0.33 \\ 0.33 & 5 & 1 & 3 \\ 0.2 & 3 & 0.33 & 1 \end{bmatrix}$

From the ordinary matrix *S*, we take its ordinary fuzzy eigenvalues and results are obtained from MATLAB eigenvalues are below:

For the left side

$$\begin{matrix}(\lambda_1)_l^0 = 3.389\\ (\lambda_2)_l^0 = 0.157\\ (\lambda_3)_l^0 = 0.1576\\ (\lambda_4)_l^0 = 0.2957\end{matrix};\quad (u_1)_l^0 = \begin{bmatrix}0.889\\0.095\\0.407\\0.185\end{bmatrix}\quad \begin{matrix}(\lambda_1)_l^1 = 4.1042\\ (\lambda_2)_l^1 = 0.0027\\ (\lambda_3)_l^1 = 0.0027\\ (\lambda_4)_l^1 = 0.0987\end{matrix};\quad (u_1)_l^1 = \begin{bmatrix}0.889\\0.086\\0.411\\0.184\end{bmatrix}$$

For the right side

$$\begin{matrix}(\lambda_1)_r^0 = 5.0535\\ (\lambda_2)_r^0 = 0.2176\\ (\lambda_3)_r^0 = 0.2176\\ (\lambda_4)_r^0 = 0.6183\end{matrix};\quad (u_1)_r^0 = \begin{bmatrix}0.879\\0.087\\0.427\\0.193\end{bmatrix}\quad \begin{matrix}(\lambda_1)_r^1 = 4.1042\\ (\lambda_2)_r^1 = 0.0027\\ (\lambda_3)_r^1 = 0.0027\\ (\lambda_4)_r^1 = 0.0987\end{matrix};\quad (u_1)_r^1 = \begin{bmatrix}0.889\\0.086\\0.411\\0.184\end{bmatrix}$$

Finally, it should notice that: finding the (fuzzy) eigenvalues is generally the most basic and main element in order to apply and utilize the methods of Factor Analysis. However, a complete fuzzification of the Correspondence Analysis as a mainly geometrical method, it require in addition of course a suitable geometrical fuzzy representation.

Conclusion

In this article we represent that: for a two-way ($n \times p$) fuzzy contingency table or fuzzy matrix with nonnegative TFN entries, the associated fuzzy correspondence analysis - fuzzy matrix *S* has TFN fuzzy eigenvalues. The procedure of the presented method is obviously easy to implement on the computer, finding, thus, the TFN-fuzzy eigenvalues of FCA. However on the contrary they do not exist always corresponding fuzzy eigenvectors, except for some simple case, as for $p = 2$. Therefore, for this fuzzy eigenvector problem of FCA, we must look for various other suitable solutions (as e.g. by a special centralization of some convenient values of each eigenvector).

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Xülasə

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Uyğunluq analizi vasitəsilə personalın seçilməsi üçün qeyri-səlis məxsusi ədəd və qeyri-səlis məxsusi vektorun hesablanması üsulunun tətbiqi

Qeyri-səlis AHP qeyri-səlis iyerarxik problemləri həll etmək üçün nəzərdə tutulmuşdur. Alternativlər arasında seçim edildikdə uyğunluq meyarının müəyyən olunmaması münaqişəli nəticələrə gətirir. Bu məqalədə müxtəlif alternativlər üçün kriteriyalar seçilir və bu kriteriyalar arasında uyğunluq analizi aparılır. Bu meyarlar qeyri-səlis ədədlər əsasında formalaşdırılır və bu alternativlərin üstünlük dərəcəsini müəyyən etmək üçün məxsusi ədədlər və məxsusi vektorlar hesablanır.

Açar sözlər: qeyri-səlis ədəd, uyğunluq analizi, qeyri-səlis matris, qeyri-səlis məxsusi ədəd, çoxmeyarlı, qeyri-səlis məxsusi vektor.

Резюме

Алиева К.Р., Гардашова Л.А., Довлатова Х.Д., Мехдиев Н.Ш.

Применение метода расчета нечетких собственных значений и нечетких собственных векторов для анализа соответствия в отборе персонала

Процесс нечеткой аналитической иерархии (FAHP) был разработан для решения неточных иерархических задач. Отсутствие последовательности в принятии решений может привести к противоречивым выводам. Трудно обеспечить последовательное попарное сравнение. В этой статье у нас есть выбор локальных и глобальных приоритетов альтернативных решений. Эти собственные значения определяются с использованием ожидаемых значений нечетких чисел и их произведений. Эти нечеткие собственные значения и собственные векторы в дальнейшем применяются для ранжирования альтернатив.

Ключевые слова: нечеткие числа, анализ соответствия, нечеткая матрица, собственное значение, многокритериальный собственный вектор.