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Estimation of the consistency in fuzzy analytic hierarch process for ranking of alternatives

Informatics and automatics

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Fuzzy Analytic Hierarch Process (FAHP) was developed to solve imprecise hierarchical problems. The absence of consistency in decision making can guide to inconsistent inferences. It is difficult to ensure a consistent pair-wise comparison. In this paper, we present a method based on the eigenvalue and eigenvector of the fuzzy comparison matrices for determination of basic focuses of decision alternatives. These eigenvalues are defined using expected values of fuzzy numbers and their products. These fuzzy eigenvalues and eigenvectors are in further procedure applied for ranking the alternatives.

Key words: fuzzy numbers, AHP, consistency index, eigenvalue, eigenvector, multicriteria, consistency ratio.

Introduction

Multi-attribute decision making (MADM) addresses the problem of choosing an optimal selection that has been highest degree of satisfaction from a set of alternatives that are characterized by different attributes. FAHP is expansion of Analytic Hierarch Process (AHP) and was created to solve imprecise hierarchical problems. As part of AHP method, a consistency determination is need to define inconsistency matrix. But, the abstention of consistency in decision making can lead to inconsistent conclusions. Therefore, the purpose of this research is to state a linear goal programming model and analysis for ordinal consistency of fuzzy complementary judgment matrix problem. Lamata and Pelaez [1] defined the Consistency Index (CI) of a matrix using the average of the consistency index of the matrix triplets. Li and Ma [2] developed a model that can assist on marking a consistent decision and used Gower plots to judge the ordinal consistency graphically. Basile and Dapuzzol [3] used the complete strict simple order to judge the ordinal consistency of judgment matrix. Luo [4] studied the revising method of judgment matrix and considered that ordinal consistency was the prerequisite of ordinal consistency. Preference relations are the most common representation of information used for solving decision making problems due to their effectiveness in modeling processes. These preference relations can be categorized into multiplicative preference relations [5, 6], fuzzy preference relations [5, 7, 10] and linguistic preference relations [11, 15]. Zhu et al. [16] demonstrated that consistent analysis should be based on ordinal consistency. On the basis of AHP consistent judgment matrix, Wang and Guo [17] directly determine the ideal priority vector by a general formula for solving the fuzzy judgment matrix priority. Zhang et al. [18] proposed a method through the no-transitive rout number and no-transitive route contribution number for solving the fuzzy judgment matrix without ordinal consistency.

Preliminaries

Since Saaty's AHP method is based on determining eigenvalue and eigenvectors of the fuzzy matrix $\tilde{F}$ at the appropriate hierarchical level, here is suggested one method to solve the fuzzy eigenvalue and eigenvector problem and find solutions of the system of homogenous fuzzy linear equations. The Analytic Hierarchy Process (AHP) permits for inconsistency because in making judgments people are more likely to be drastically inconsistent than drastically consistent because they cannot evaluate exactly measurement values even from a known scale and worse when they deal with intangibles (*a* is preferred to *b* twice and *b* to *c* three times, but *a* is preferred to *c* only five times) and ordinally intransitive (*a* is preferred to *b* and *b* to *c* but *c* is preferred to *a*) [19]. We can list the measurements as w_1, w_2, ..., w_n. If we compare the measurement for element *i* with the measurement of element *j*, the result of the comparison is a value expressed as the ratio between w_i and w_j. The opposite comparison, between *j* and *i*, is clearly therefore expressed as the ratio between w_j and w_i. If we accept this symmetry of comparison, the matrix created has a particularly interesting property which turns out to be useful in examining the degree of consistency when we do not have the requisite measuring tool [20].

Definition 1. Fuzzy matrix $\tilde{F}$ characterized by low, medium and upper crisp matrices. Crisp matrices F_l, F_m, F_u are obtained according to

$$\tilde{f}_{ij} = (f_{ij,l}, f_{ij,m}, f_{ij,u}) \tag{1}$$

$$A_{n\times m} = \begin{matrix} & C_1 & C_2 & \dots & C_m \\ f_1 & x_{11} & x_{12} & \dots & x_{1m} \\ f_2 & x_{21} & x_{22} & \dots & x_{2m} \\ \dots & \dots & \dots & \dots & \dots \\ f_n & x_{n1} & x_{n2} & \dots & x_{nm} \end{matrix} \tag{2}$$

Definition 2. The triangular fuzzy number, as a special type of a fuzzy set over the set of real numbers (real line) R .Obtained matrices are used for calculation of a system of fuzzy linear homogenous equations.

$$\overline{F_l} w_l + \overline{F_m} w_m + \overline{F_u} w_u - \overline{\lambda_l} w_l - \overline{\lambda_m} w_m - \overline{\lambda_u} w_u = 0 \tag{3}$$

where,

$$\begin{aligned} \overline{\lambda_l} &= 2\lambda_l + \lambda_m \\ \overline{\lambda_m} &= \lambda_l + 4\lambda_m \\ \overline{\lambda_u} &= \lambda_m + 2\lambda_u \end{aligned} \tag{4}$$

Definition 3. All the values in these equations are positive ones, crisp eigenvalues are represented by three systems:

$$\begin{aligned} \overline{F_l} w_l &= \overline{\lambda_l} w_l \\ \overline{F_m} w_m &= \overline{\lambda_m} w_m \\ \overline{F_u} w_u &= \overline{\lambda_u} w_u \end{aligned} \tag{5}$$

An important consideration in terms of the quality of the ultimate decision relates to the consistency of judgments that the decision maker demonstrated during the series of pairwise comparisons.

Definition 4. Consistency index and criterion allows the decision maker to study the consistency of each matrix in an adaptable way. Using the index and criterion that we present, the user can decide about the matrix consistency using not only the matrix entries, but also the level of consistency that the decision maker needs in this particular case. The consistency index (CI):

$$\mathrm{CI} = \frac{\lambda_{\max} - \mathrm{n}}{\mathrm{n} - 1} \tag{6}$$

The AHP provides a measure of the consistency of pairwise comparison judgments by computing a consistency ratio.

Definition 5. The consistency ratio (CR):

$$CR = \frac{CI}{RI} \tag{7}$$

Table 1. Average random consistency RI

n	1	2	3	4	5	6	7	8	9	10
RI	0.00	0.00	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49

CR is less than 0.10, it means that the chosen matrix is optimal.

Statement of the problem

The proposed method of the fuzzy AHP was applied for the personal selection problem which is a multicriteria decision making (MCDM) problem since it contains different and conflicting criteria. Suppose that an MCDM problem involves 5 criteria – C_1, C_2, C_3, C_4, C_5 and 5 alternatives – *A, B, D, E, F*. C_1 – Relevant experience; C_2 – Education; C_3 – Technical skills; C_4 – Relocation, C_5 – Communication.

Table 2. Linguistic performance rating of fuzzy numbers

Criteria	C_1	C_2	C_3	C_4	C_5
C_1	(1,1,1)	(1/5,1/3,1/1)	(1/4,1/2,1/1)	(1/5,1/3,1/1)	(1/6,1/4,1/2)
C_2	(1,3,5)	(1,1,1)	(1,2,4)	(1/5,1/3,1/1)	(1/4,1/2,1/1)
C_3	(1,2,4)	(1/4,1/2,1/1)	(1,1,1)	(1/4,1/2,1/1)	(1/4,1/2,1/1)
C_4	(1,3,5)	(1,3,5)	(1,2,4)	(1,1,1)	(1/5,1/3,1/1)
C_5	(2,4,6)	(1,2,4)	(1,2,4)	(1,3,5)	(1,1,1)

When the elements compared are expressed in the same units of measurement and we use a suitably precise measuring tool, then pairwise comparison is unnecessary. The answer to the question of how much more important one element is than the others is given simply by the combination of the measurements of all the elements. It is worth investigating the properties of the comparison matrix in the case of precise measurements. The pairwise comparison matrix for precise measurements of any number of elements is a consistency matrix. The initiator of the AHP, Thomas Saaty, discovered certain properties of this type of matrix, which allow the verification of the consistency of the measurements in situations when there is no opportunity for taking precise measurements [20].

Solution of the problem

The basic idea is that a matrix is consistent or not depending on the scope. In different cases, the decision makers required different degrees of consistency and we can represent these levels consistency in the analytic hierarchy process. One special matrix is therefore either consistent or not consistent depending on two different factors: a) a consistency index (λ_{max}); b) the level of consistency needed (α), $0 < \alpha \leq 1$. This level α provides adaptability to different scopes. In this case, we can determine if a specific matrix is a sufficiently. By using MATLAB we obtained three crisp matrices:

FL				
1	0.2	0.25	0.2	0.17
1	1	1	0.2	0.25
1	0.25	1	0.25	0.25
1.00	1	1	1	0.2
2.00	1.00	1.00	1.00	1.00

FM				
1	0.33	0.5	0.33	0.25
3	1	2	0.33	0.5
2	0.5	1	0.5	0.5
3	3.00	2.00	1	0.33
4.00	2.00	2.00	3.00	1.00

FU				
1	1	1	1	0.5
5	1	4	1	1
4.00	1	1	1	1
5	5	4	1	1
6.00	4.00	4.00	5.00	1.00

$$\overline{F_l} = 2F_l + F_m$$
$$\overline{F_m} = 2F_l + 4F_m + F_u$$
$$\overline{F_u} = F_m + 2F_u$$

Using this formula results are below:

$\overline{F_l}$				
3.0	0.7	1.0	0.7	0.6
5.0	3.0	4.0	0.7	1.0
4.0	1.0	3.0	1.0	1.0
5.0	5.0	4.0	3.0	0.7
8.0	4.0	4.0	5.0	3.0

$\overline{Fm}$				
6.0	2.5	3.3	2.5	1.7
18.0	6.0	13.0	2.5	3.3
13.0	3.3	6.0	3.3	3.3
18.0	18.0	13.0	6.0	2.5
24.0	13.0	13.0	18.0	6.0

$\overline{F_u}$				
3.0	2.3	2.5	2.3	1.3
13.0	3.0	10.0	2.3	2.5
10.0	2.5	3.0	2.5	2.5
13.0	13.0	10.0	3.0	2.3
16.0	10.0	10.0	13.0	3.0

With MATLAB software program eigenvalues are obtained

$\overline{\lambda_l}$ =11.4 $\overline{\lambda_m}$ =34.01 $\overline{\lambda_u}$ =24.6

By using linear homogenous equations λ_l, λ_m, λ_u are founded

$$\lambda_l = 3.03$$
$$\lambda m = 5.34$$
$$\lambda u = 9.63$$

On this basis, we can construct indicators showing the consistency of the expert's estimates. For evaluations of consistency, T. Saaty [20] proposes the following solution:

$$CI = \frac{5.34 - 5}{5 - 1} = 0.085$$
$$CR = \frac{0.085}{1.12} = 0.075$$
$$0.075 \le 0.10$$

The consistency index (CI) refers to the average of the remaining solutions of the characteristic equation for the inconsistent matrix *A*. This index increases in proportion to the inconsistency of the estimates. In a decision problem it is common practice to ask the decision maker to revise his judgments until a value of CR smaller than 0.1 is reached.

Conclusion

The AHP method is very useful method for MCDM process in which there is hierarchy between criteria, sub-criteria and alternatives. This process proposed more complete, flexible, realistic results specifically for the decision criteria that have qualitative nature. Applying MATLAB software program the following eigenvalues for the matrix C_{ij} are λ_l, λ_m, λ_u. For the C_{ij} matrix consistency index and consistency ratio is very small, from this results *C* matrix is very consistent and may be accepted.

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Xülasə

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Alternativlərin ranqlaşdırılmasında analitik iyerarxik prosesinin uyğunluq meyarının qiymətləndirilməsi

Qeyri-səlis AHP qeyri-səlis iyerarxik problemləri həll etmək üçün nəzərdə tutulmuşdur. Qərar qəbul edilməsində uyğunluğun olmaması münaqişəli nəticələrə gətirib çıxara bilər. Davamlı cüt müqayisəni təmin etmək çətindir. Məqalədə alternativ həllər üçün yerli və qlobal prioritetlərimiz var. Bu məxsus ədədlər qeyri-səlis ədədlərin və onların məhsullarının gözlənilən qiymətlərindən istifadə edərək müəyyən edilir. Bu qeyri-səlis məxsusi ədəd və məxsusi vektorlar alternativlərin sıralamasında da tətbiq edilir.

Açar sözlər: qeyri-səlis ədədlər, AHP, uyğunluq indeksi, məxsusi ədəd, məxsusi vektor, çoxmeyarlı, uyğunluq nisbəti.

Резюме

Алиева К.Р., Мехдиев Н.Ш.

Оценка критериев соответствия процесса аналитической иерархии при ранжировании альтернатив

Процесс нечеткой аналитической иерархии (FAHP) был разработан для решения неточных иерархических задач. Отсутствие последовательности в принятии решений может привести к противоречивым выводам. Трудно обеспечить последовательное попарное сравнение. В этой статье у нас есть выбор локальных и глобальных приоритетов альтернативных решений. Эти собственные значения определяются с использованием ожидаемых значений нечетких чисел и их произведений. Эти нечеткие собственные значения и собственные векторы в дальнейшем применяются для ранжирования альтернатив.

Ключевые слова: нечеткие числа, AHP, индекс непротиворечивости, собственное значение, собственный вектор, многокритериальный, коэффициент непротиворечивости.